IOP Publishing Superconductor Science and Technology
Supercond. Sci. Technol. **21** (2008) 082003 (3pp) doi:10.1088/0953-2048/21/8/082003

RAPID COMMUNICATION

Superfluid density in a superconductor with an extended d-wave gap

Ruslan Prozorov

Ames Laboratory and Department of Physics and Astronomy, Iowa State University, Ames, IA 50011, USA

E-mail: prozorov@ameslab.gov

Received 19 March 2008, in final form 30 May 2008
Published 16 June 2008
Online at stacks.iop.org/SUST/21/082003

Abstract
The superconducting gap and the superfluid density are calculated for the extended d-wave gap suggested by angle-resolved photoemission spectroscopy (ARPES) measurements in electron-doped superconductors as well as underdoped $Bi_2Sr_2CaCu_2O_{8+x}$ (BSCCO2212). It is shown that experimental superfluid density may agree with such a gap, but full-temperature range analysis is required. With additional information on the $\Delta(0)/T_c$ ratio, this opens up the possibility of deciding whether a non-monotonic or higher harmonic spectral gap, as seen by ARPES, is a superposition of a regular d-wave gap and an unrelated pseudogap that does not contribute to superconductivity or that the extended gap is a real superconducting gap. This paper is also an erratum to earlier calculations published in Prozorov and Giannetta (2006 *Supercond. Sci. Technol.* **19** R41–67) where the wrong sign was used for the non-monotonic gap, altering the conclusions rather drastically.

(Some figures in this article are in colour only in the electronic version)

The analysis of temperature-dependent superfluid density in superconductors is a powerful tool for examining pairing symmetry [1]. Although it does not detect the phase of the order parameter, this bulk probe is very sensitive to the gap structure on the Fermi surface. Therefore it can be used to verify the conclusions provided by the gap-mapping techniques such as angle-resolved photoemission spectroscopy (ARPES) or directional tunneling. ARPES experiments suggest that the gap variation on the Fermi surface in high-T_c cuprates should take into account long-range interactions, which leads to the inclusion of the higher harmonic, consistent with the d-wave pairing. Such an extended d-wave gap can be written as

$$\begin{aligned}\Delta(T,\varphi) &= \Delta(T)\, g(\varphi)\\ &= \Delta(T)\left[B\cos(2\varphi) + (1-B)\cos(6\varphi)\right]\end{aligned} \quad (1)$$

with $B = 0.88$ for underdoped ($T_c = 80$ K) $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Bi2212) [2]; $B = 1$ for overdoped Bi2212 ($T_c = 80$ K) [2]; $B = 0.78$ for optimally doped ($T_c = 35$ K) $(Bi, Pb)_2(Sr, La)_2CuO_{6+\delta}$ (Bi2201) [3]; and $B = 1.43$ for optimally doped ($T_c = 26$ K) electron-doped cuprate $Pr_{0.89}LaCe_{0.11}CuO_4$ (PLCCO) [4]. For a pure s-wave, $g(\varphi) = 1$; for pure d-wave, $B = 1$ and $g(\varphi) = \cos(2\varphi)$, as shown in figure 1. Note an unusual value for B in the case of PLCCO that exceeds 1 and, as we show below, results in a quite different gap topology and resulting superfluid density as a function of temperature.

In our recent review [1], the superfluid density was calculated for the gap in the form of equation (1), intending to illustrate that, according to equation (1), the case of $B = 1.43$ gives $g(\varphi) = 1.43\cos(2\varphi) - 0.43\cos(6\varphi)$. However, the calculations were performed for $g(\varphi) = 1.43\cos(2\varphi) + 0.43\cos(6\varphi)$ with '+' instead of '−', changing all results rather dramatically. The intent of this paper is to point out this error and calculate the gap and the superfluid densities for several cases where equation (1) was found to be relevant. This provides a basis for direct comparison between ARPES and penetration depth measurements.

Although the Eilenberger quasi-classical formulation is suitable for a general anisotropic Fermi surface, gap symmetry, and scattering [5], it involves slowly converging Matsubara sums. Results utilizing the Eilenberger approach with arbitrary scattering will be published elsewhere [7]. Here we follow a

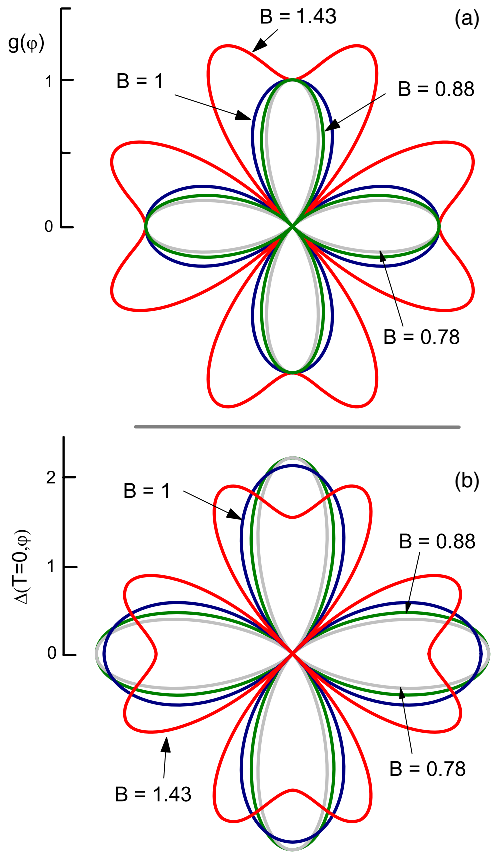

Figure 1. (a) Angular part of the superconducting gap for various cases discussed in the text. (b) Variation of the gap magnitude on the Fermi surface for each case.

simpler integral form of the quasi-classical approach suggested by Chandrasekhar and Einzel [6]. The total superfluid density in the clean case is given by,

$$n_{s,i} = \frac{k_F}{4\pi^3}\oint_{FS} \mathrm{d}S_k \times \left[\left(\frac{\mathbf{v}_F\otimes\mathbf{v}_F}{v_F^2}\right)_{ii}\left(1+2\int_{\Delta_k}^{\infty}\mathrm{d}E\,\frac{\partial f}{\partial E}\frac{N(E)}{N(0)}\right)\right] \tag{2}$$

where $n_{s,i}$ is a component of the superfluid density corresponding to the particular diagonal component of the tensor product of Fermi velocities $\mathbf{v}_F \otimes \mathbf{v}_F$, k_F is the magnitude of the Fermi wavevector, f is the Fermi–Dirac distribution function, $N(E)$ is the density of states and $E = \sqrt{\varepsilon^2 + \Delta_k^2}$ is the quasi-particle energy spectrum. The integration is carried over the entire Fermi surface. Assuming a cylindrical Fermi surface appropriate for cuprates, the normalized superfluid density in the ab-plane is reduced to

$$\rho_s = \left[\frac{\lambda(0)}{\lambda(T)}\right]^2 = 1 - \frac{1}{2\pi T}\int_0^{2\pi}\mathrm{d}\varphi\cos^2(\varphi) \times \left[\int_0^{\infty}\mathrm{d}\varepsilon\cosh^{-2}\left(\frac{\sqrt{\varepsilon^2+\Delta(\varphi)^2}}{2T}\right)\right]. \tag{3}$$

To calculate the superfluid density, the temperature dependence of the gap magnitude has to be determined from

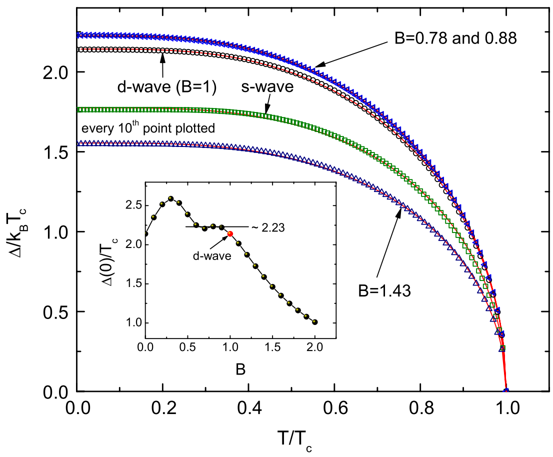

Figure 2. Amplitude of the superconducting gap, $\Delta(T)$ (see equation (1)) as a function of temperature calculated from numerical solutions of the gap equation, equation (4). Inset: $\Delta(0)/T_c$ versus B.

Table 1. Experimental and fitting parameters for various gaps discussed in the text.

System	Gap	T_c (K)	$\frac{\Delta(0)}{T_c}$-calc.	$\frac{\Delta(0)}{T_c}$-exp.	δ	c
Generic	s-wave	—	1.76		1.77	1.00
OD BSCCO2212	d-wave	80	2.14	3.92	2.15	1.33
Opt BSCCO-2201	$B = 0.78$	35	2.23	13.26	2.22	1.33
UD BSCCO2212	$B = 0.88$	80	2.23	6.24	2.23	1.34
PLCCO	$B = 1.43$	26	1.55	0.85	1.55	0.72

the self-consistent gap equation,

$$\int_0^{2\pi}\mathrm{d}\varphi\,\mathrm{d}\varepsilon\, g(\varphi)^2\left[\frac{\tanh\left(\frac{1}{2T}\sqrt{\varepsilon^2+[\Delta(T)\,g(\varphi)]^2}\right)}{\sqrt{\varepsilon^2+[\Delta(T)\,g(\varphi)]^2}} - \frac{1}{\varepsilon}\tanh\left(\frac{\varepsilon}{2}\right)\right] = 0. \tag{4}$$

A very good approximation for $\Delta(T)$ in the entire temperature range is given by

$$\Delta(T) = \delta\tanh\left(\frac{\pi}{\delta}\sqrt{c\left(\frac{T_c}{T}-1\right)}\right). \tag{5}$$

Equation (4) was solved together with equation (1) numerically using Matlab and Mathematica (both yielded the same results, but Mathematica was significantly slower) and then fitted the obtained curves to equation (5), as shown in figure 2. Fit parameters are summarized in table 1.

For reference, the ratio $\Delta(0)/T_c$ has been computed for a wider variation of $0 \leqslant B \leqslant 2$ and is plotted in the inset to figure 2. The curve is surprisingly non-monotonic for $B < 1$, but varies smoothly for $B \geqslant 1$.

Based on the data from ARPES measurements, we estimate the $\Delta(0)/T_c$ ratio and compare it with the values calculated from equation (4), shown in table 1. While the values for OD BSCCO2212 (83%) and PLCCO (45%) are moderately different, the systems with $B < 1$ show too much of a disagreement (e.g. 1100% for Opt BSCCO2212) and it

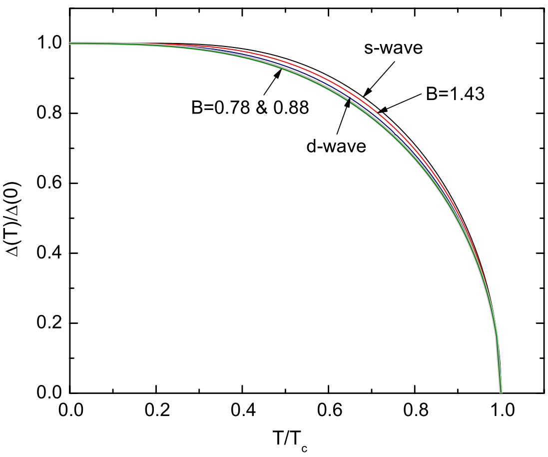

Figure 3. Normalized temperature-dependent part of the gap versus reduced temperature.

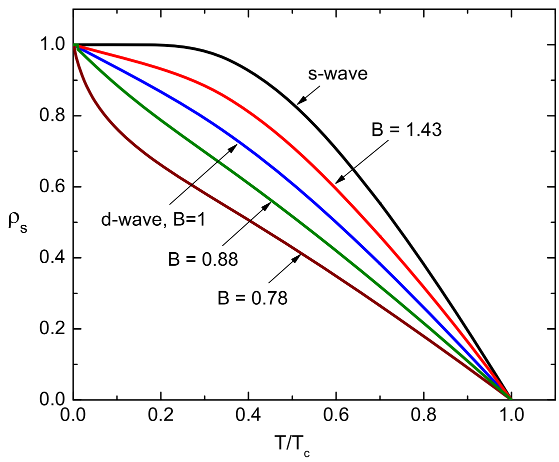

Figure 4. Superfluid density as a function of temperature calculated for different gaps discussed in the text.

is unlikely that this discrepancy can be explained by different coupling strength, for example, or other effects. Overall, the temperature-dependent part of the gap does not change much for $0.5 < B < 1$. This is clearly seen when the normalized gap, $\Delta(T)/\Delta(0)$, is plotted (figure 3).

Finally, the primary result is the temperature-dependent superfluid density calculated with the extended gap in the form of equation (1) as if it all contributes to superconductivity. Figure 4 shows the superfluid densities for different gaps. All calculations are performed in the clean limit. Whilst s-wave, d-wave and $B = 1.43$ look conventional and close to those reported in the literature [8, 9] (allowing for the uncertainty in $\lambda(0)$), the $B < 1$ gaps produce concave superfluid density that has not been reported in high-T_c cuprates yet. Although it may in principle exist, such behavior is unlikely and implies that the spectral gap measured by ARPES in these materials is a combination of the usual ($B = 1$) superconducting gap and a pseudogap which does not contribute to the condensate. However, the pseudogap affects the spectral density of the quasi-particles and this has to be taken into account [3] when calculating the superfluid density. Precise measurements of the temperature-dependent penetration depth along with its zero-temperature value are needed to estimate the superfluid density reliably. This experimental work will be reported elsewhere.

In conclusion, the temperature dependence of the superconducting gap and the superfluid density have been calculated in the clean limit weak-coupling BCS (Bardeen, Cooper and Schrieffer) approximation for the extended d-wave gap where the next angular harmonic is taken into account. For $B \geqslant 1$ the superfluid density exhibits the usual convex behavior, whereas for $B < 1$ the behavior is concave. While the question of the pseudogap in electron-doped cuprates is still open, our results suggest that the behavior is quite different from the hole-doped counterparts, and the non-monotonic gap with $B = 1.43$ produces superfluid density that would be difficult to discard based only on the analysis of the penetration depth measurements, and additional information about the $\Delta(0)/T_c$ is required.

Acknowledgments

I thank Hubertus Luetkens for pointing out the wrong sign in my original calculations. Discussions with Vladimir Kogan and Adam Kaminski are greatly appreciated. Work at the Ames Laboratory was supported by the Department of Energy-Basic Energy Sciences under contract no. DE-AC02-07CH11358. I also acknowledge support from US National Science Foundation (NSF) grant number DMR-06-03841 and the Alfred P Sloan Foundation.

References

[1] Prozorov R and Giannetta R W 2006 *Supercond. Sci. Technol.* **19** R41–67
[2] Mesot J *et al* 1999 *Phys. Rev. Lett.* **83** 840
[3] Kondo T, Takeuchi T, Kaminski A, Tsuda S and Shin S 2007 *Phys. Rev. Lett.* **98** 267004
[4] Matsui H, Terashima K, Sato T, Takahashi T, Fujita M and Yamada K 2005 *Phys. Rev. Lett.* **95** 017003
[5] Eilenberger G 1968 *Z. Phys.* **214** 195
[6] Chandrasekhar B S and Einzel D 1993 *Ann. Phys., Lpz.* **2** 535
[7] Kogan V G and Prozorov R 2008 in preparation
[8] Prozorov R, Giannetta R W, Fournier P and Greene R L 2000 *Phys. Rev. Lett.* **85** 3700
[9] Snezhko A, Prozorov R, Lawrie D D, Giannetta R W, Gauthier J, Renaud J and Fournier P 2004 *Phys. Rev. Lett.* **92** 157005